Tubular Sculptures

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Abstract

This paper reviews ways in which many artists have constructed large sculptures from tubular elements, ranging from single cylinders to toroidal or knotted structures, to assemblies of a large number of bent tubes. A few parameterized generators are introduced that facilitate design and evaluation of a variety of such sculptural forms.

1. Introduction

Artists like Charles O. Perry have been able to build very large scale sculptures filling volumes of more than 30 feet in diameter at an affordable price by assembling pre-cut and bent tubular pieces. Stellar examples are *Eclipse* in the Hyatt Regency lobby in San Francisco, or *Equinox* at the Lincoln Center, Dallas, Texas (Fig.1a). But even much smaller assemblies of tubular elements can make very attractive sculptures. At the small end of this spectrum we find sculptures by Max Bill, e.g., *Assembly of three equal cylinders* (Fig.1b) [2], or the elegant tubular loops by José de Rivera (Fig.1c) [4]. Additional "minimal sculptures" will be discussed in Section 3.

Figure 1: *(a) Charles O. Perry: "Equinox;" (b) Max Bill: "Assembly of 3 equal cylinder;" (c) José de Rivera: "Construction #35."*

With so many diverse ways of forming attractive sculptures from tubular elements, it seems worthwhile to try to compile an organized overview over the many possibilities and approaches used, and to explore in which ways computer-aided tools may be helpful to create additional, and potentially more complex, artistic structures. In addition, I have a personal, nostalgic reason to write this paper on *Tubular Sculptures*. It was the influence of Frank Smullin who started me a quarter century ago on this path of exploring "Artistic Geometry" at the intersection of computer science, mathematics, and sculpture.

2. Inspiration by Frank Smullin (1943 – 1983)

On June 30, 1981, Frank Smullin gave a keynote talk at the 18th Design Automation Conference in Nashville, Tennessee [19]. He showed pictures of his tubular sculptures, often celebrating simple knots (Fig.2a). He explained how he had created a program running on his Apple II computer to help him design these sculptures and to calculate the elliptical cut lines that resulted at the angled junction between two or more tubular elements. His program produced simple outline drawings, which he then colored in by hand (Fig.2b). In this talk he extolled the merits of the *Granny-knot*, which has an artistically much more interesting, 3-dimensional structure than the functionally preferred, but much flatter *Square-knot*. When he mentioned that in the *Granny-knot* the four ends of the cropped "shoe laces" would stick out in the 4 tetrahedral directions (Fig.2c), something clicked in my brain. Four arms sticking out in tetrahedral directions reminded me of a carbon atom and of the way such atoms readily assemble into a diamond lattice. I immediately saw the possibility of assembling a large number of *Granny-knots* in the same way into a *Granny-Knot Lattice* (Fig.2d). I left the conference with the firm goal that I wanted to create such a structure, and that I would use the emerging power of computer graphics to help me in that venture.

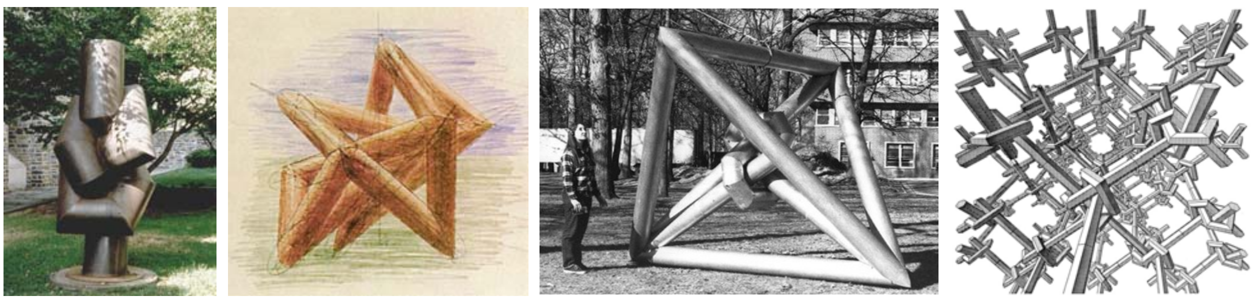

Figure 2: *(a) Sculpture by Smullin; (b) Smullin's design program; (c) Smullin: "Tetra-Granny;" (d) Séquin: "Granny-Knot Lattice."*

Over the following twelve months, Paul Strauss, a one-year-on-campus Master student from Bell Labs, developed a first version of a rendering program, called Berkeley *UNIgrafiX*, which was able to render such structures with proper hidden-surface elimination; thus there was no need to shade in the visible surfaces by hand. In parallel I developed a program *MakeWorm* that created the properly mitered and joined prismatic segments that would form tight, knotted or linked structures, and within a few months I had a dramatic visualization of the *Granny-Knot Lattice* displayed in the {1 1 0} direction of the diamond lattice structure to reveal the well-known channels that run in that particular direction (Fig.2d).

Figure 3: *Tubular sculpture models by Séquin: (a) "Standing Overhand Knot," (b) "Mitered Junction Models," (c) "Minimal Sticks Trefoil," (d) "Tetra-Tangle."*

Inspired by Smullin's sculptures, I also used this emerging programming environment to design some free-form knot models (Fig.3a), made from the 4-inch diameter cardboard tubes that carried the 3-foot wide Versatec printer paper. In that effort I used the mitering techniques explained by Smullin, and made many tutorial models (Fig.3b), which I then used in my class on *Creative Geometric Modeling*. Figure 3c shows the 6-stick realization of the trefoil knot with shortest total axis length, which was obtained with a simple optimization procedure. I also was proud to discover a tight, highly symmetrical linkage of four mutually linked equilateral triangles (Fig.3d), but subsequently discovered that this link and many other, much more ingenious ones had already been described in the book *Orderly Tangles* [10].

3. Minimal Geometries – Cylinders

Trying to find a systematic approach to a classification of tubular sculptures, we start with the simplest tubular element: a straight, circular cylinder. In this raw form it shows up as an architectural element, e.g. in *La Défense*, near La Garenne-Colombes, France (Fig.4a) and in the chapel at MIT in Cambridge, MA (Fig.4b). When a single cylinder is used as a stand-alone sculpture, it normally is given some artistic enhancement. This may be achieved by painting its surface as in Max Bill's *Painting in form of a column* (Fig.4c) or by cutting off its ends in some special way as in Bill's *Cylinder as a right-angled volume* (Fig.4d). Alternatively only part of the cylinder surface may be retained, as in John Goodman's *Unicorn Spirit* (Fig.4e).

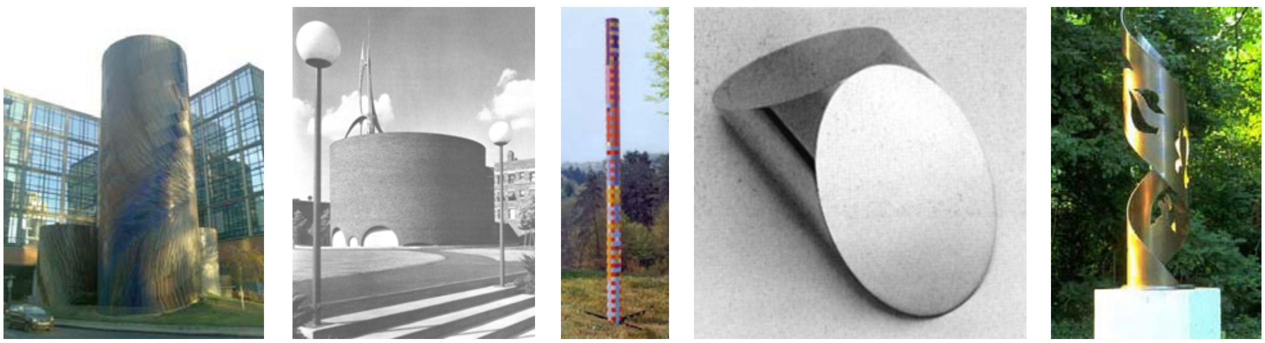

Figure 4: *(a) Cylinder in La Défense; (b) Chapel at MIT; (c) Bill: "Painting in form of a column;" (d) Bill: "Cylinder as a right-angled volume;" (e) Goodman: "Unicorn Spirit."*

Many large-scale sculptures are formed by groups of simple un-decorated cylinders, e.g., *FUNtain Hydraulophone* at the Ontario Science Centre [7] or Roger Berry's *Rising Wave* at Oyster Bay [1].

Figure 5: *(a) Hart: "72 Pencils – CMYK;" (b) Séquin: "Skewed Tangle of 12 Cylinders;" (c) Snoeyink: "30 Aluminium Tubes;" (d) Snelson: "Needle Tower."*

When cylinders are packed in tight, tangled configurations, we obtain very intriguing geometrical configurations. An example of such a cylinder packing, well known to Bridges attendees, is George Hart's *72 Pencils – CMYK* (Fig.5a) [8]. Figure 5b shows a tangle of 12 cylinders derived from the TetraTangle structure (Fig.3d). A more elaborate version of such a skewed arrangement has been constructed by Jack Snoeyink; it is made from 30 identical aluminum tubes, colored and grouped as five twisted, interlocking tetrahedral frames (Fig.5c). This sculpture celebrates a research result that shows that this arrangement, made from simple straight cylinders, cannot be divided into two groups that can then be separated with a rigid-body motion [20]. In the context of cylinders used as sculptural elements, I would also like to reference the *tensegrity* structures pioneered by Kenneth Snelson (Fig.5d), which may be perceived as collections of cylinders seemingly floating free in space.

4. Tori and Chain Links

If we allow bending the tubular elements, then the simplest most regular shape we can form is the torus. Tori have been celebrated by many artists in many different ways. In the science museum in Vienna there is an archeological artifact in bronze representing a simple toroidal shape of more than a foot in diameter (Fig.6a), dating back about 5000 years! Given more than one torus – they just want to be interlinked. A beautiful tight linking of two tori has been created by John Robinson with *Bonds of Friendship* in Sydney Cove, Australia (Fig.6b). Three interlocking tori can be seen in a giant inflatable sculpture *Torus! Torus!* erected by Joseph Huberman (Fig.6c) [11].

Figure 6: *(a) 5000 year old bronze torus; (b) Robinson: "Bond's of Friendship;" (c) Huberman: "Torus! Torus!"*

Helaman Ferguson even has accomplished the feat of interlocking two tori made of different types of stone (Fig.7a). – Of course, they are not really complete tori, but rings with beveled gaps, depicting the self-intersecting surfaces of two Klein bottles; these two open chain links can then be assembled with a clever twisting maneuver. Stretched toroidal loops can readily be linked into longer chains. Tight-fitting plastic chain links have been assembled into deformable chains and then modeled into catenoids standing upside down in my sculpture model *Defying Gravity* (Fig.7b). Such tubular ovals are also the basis for a configuration of the Borromean rings cast by Alex Feingold (Fig.7c). In 1999 I created a Borromean tangle with five oval loops (Fig.7d). Interlinking ever more tubular loops in ever more intricate ways leads to a plethora of interesting configurations, as depicted in *Orderly Tangles* [10]. Some of these links would make quite dramatic large-scale sculptures. On a small scale (Fig.3a in [15]) they make bouncy dog toys or colorful "Nobbly Wobblys" [5].

Figure 7: *(a) Ferguson: "Four Canoes;" (b) Séquin: "Defying Gravity;" (c) Feingold: "Borromean rings;" (d) Séquin: "Borromean Tangle 5."*

5. Ruled Surfaces and Iterated Hula-Hoops

The simple geometrical elements that made up the sculptures discussed so far maintain some individuality. Now we look at configurations where the individual cylinder or torus has little meaning, and where it is a relatively large collection of such elements that then create the dominant form of the sculpture. As a first example we look at a large collection of long, skinny cylinders to form a more complex geometrical shape. Such geometry is reminiscent of *String Art* and of many mathematical models depicting hyperbolic paraboloids and other ruled surfaces. Tubular elements forming such ribbed surfaces are particularly useful when "transparency" is required. This is the case when we try to depict mathematical models with intersecting surfaces and would like to provide a look to the inside, to see interesting features such as triple points. Figure 8a and b show tubular models of a hemi-cube and of a hemi-dodecahedron. These are non-orientable, generalized polyhedra (or 2D cell-complexes) that can be obtained by taking surface elements of the cube or of the dodecahedron and identifying antipodal points, edges, and faces. Thus my model of the hemi-cube has just three bilinearly warped "squares," and the hemi-dodecahedron is composed of six (differently colored) warped pentagons forming a tetrahedral structure with six angled edges. The latter is the basic building block of the 4-dimensional 57-Cell [17].

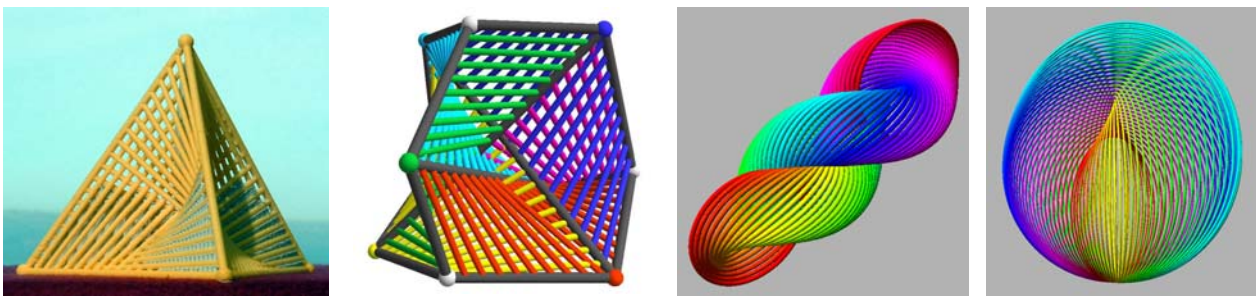

Figure 8: *Non-orientable 4-dimensional cells: (a) hemi-cube, (b) hemi-dodecahedron. Iterated circles: (c) twisted goblet, (d) cross-cap model of the projective plane.*

Similarly, a large assembly of skinny tori can produce interesting shapes. Just rotating such a "hula-hoop" around one of its diameters and simultaneously sweeping it along that diameter creates a twisted goblet (Fig.8c). If instead the hula-hoop is iteratively scaled from size 1.0 to size 2.0 and back to size 1.0 while we rotate the hoop through 180°, we obtain a rendering of a cross-cap – a model of the projective plane.

These iterative constructions, which are particularly amenable to generation with a computer program, can be generalized to *Ribbed Surfaces*, where the iterated ribs need not be straight cylinders nor tori, but may be a sequence of procedurally specified curves. One versatile approach first defines two *Guide Rails* and then suspends a collection of equally spaced Bézier curves or circular arcs between them.

6. Perry's Ribbed Sculptures

Charles O. Perry has created several large *Ribbed Sculptures* [14] that can be modeled with this paradigm; and his sculptures were the inspiration for this work and for this paper. A sculpture that is particularly fascinating to the mathematical mind is *Solstice* (Fig.9a) because its generating principle is so simple and elegant, yet the result is intriguing and spectacular in the variety of vistas it offers from different angles. *Solstice* has a thick rail that forms a (3,2) torus knot embedded in the surface of a fat virtual torus with a rather small central hole. Each minor circle of that torus has 3 symmetrically located points, 120° apart, through which the thick rail passes. These three points are connected with three thin ribs that form a "hyperbolic" triangle with inward-bending concave sides (Fig.9e). This triangular cross section now twists through 240° as it travels once around the major loop of the torus. (Helaman Ferguson has created a similar geometry in a bronze cast called *Umbilic Torus NC*; however, in his sculpture the twist of the triangular cross section is only 120°.)

Understanding the mathematics behind *Solstice* allows us to create a close computer emulation (Fig.9b). This generator program, written by James Hamlin, can now be parameterized so that we can readily change the number of ribs and their diameter or curvature to fine-tune the look and feel of the sculpture. But we can also introduce more dramatic changes by altering the integer parameters (p,q) that define the torus knot formed by the supporting rail. Figure 9c shows a sculpture that results from a (4,3) torus knot with all other parameters left unchanged. Figure 9d is yet another variation, based on a (2,3) torus knot; however, in this sculpture the ribs no longer lie in the planes of the minor circles of the torus, instead they are spiraling a significant distance forward or backward around the major loop of the torus. An additional parameter in the program allows us to control the amount of this angular offset around the major circle of the torus; in Figure 9d this value was –45 degrees.

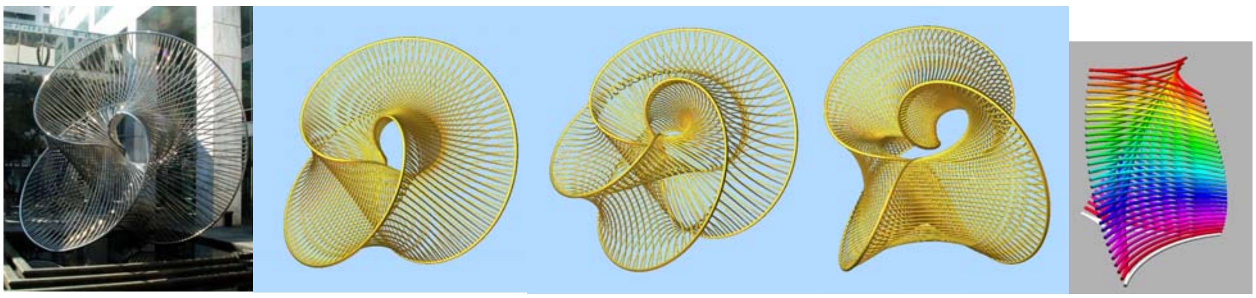

Figure 9: *"Solstice": (a) Perry's sculpture; (b) computer emulation; (c) (4,3) torus knot variation; (d) (2,3) torus knot variation. (e) Principle of swept 'hyperbolic' triangle.*

Early Mace (Fig.10a) is another sculpture by Charles O. Perry that uses ribs in the shape of circular arcs connecting two guide rails. In this case the rails are two pairs of (almost) great semicircles on an invisible sphere, held together by two small semicircles at both ends. The ribs form inwards-bending quarter arcs. Figure 10b shows an emulation of this sculpture with a program written by James Hamlin. Figures 10c and 10d then demonstrate what happens when the ribs are straightened and finally bent outward to follow the surface of the sphere.

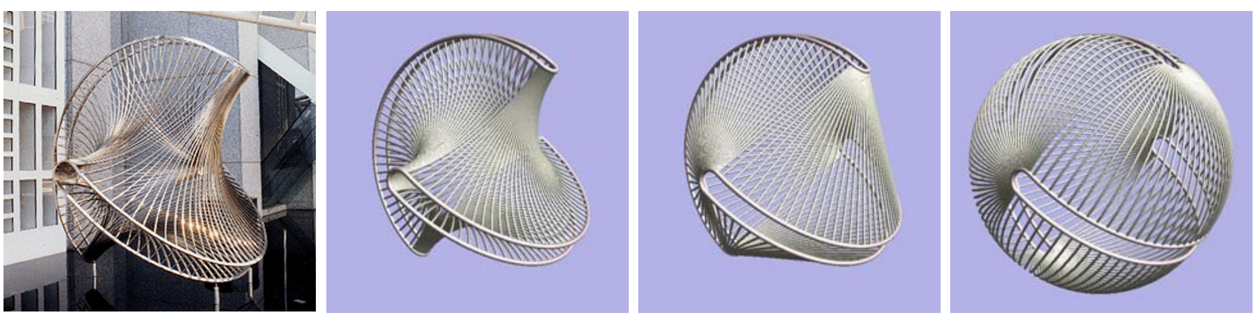

Figure 10: *"Early Mace": (a) Perry's sculpture; (b) computer emulation; (c,d) rib variations.*

7. Generalized Sweeps

Tubular elements provide broad artistic freedom of expression if free-form guide curves are used for the sweeps of circular or of more elaborate cross sections. Very simple free-form shapes have appeared as icons on musical album covers (Fig.11a) [13] or have been used by artists such as José de Rivera for 3D metal sculptures (Fig.11b). Free-form shapes typically need customized bending of tubes, and this may require tools and methods not readily accessible to many people. In 1982 Richard Zawitz introduced the *Tangle®-Toy*, a series of pipe segments, each curbed through 90°, connected into a loop and able to pivot at each joint (Fig.11c). This pliable assembly now allows anybody to become an instant tubular artist and to create attractive free form sculptures (Fig.11d).

Figure 11: *(a) "Tubular Bell" album cover; (b) de Rivera: "Construction #5B." (c) Zawitz: "Museum Tangle;" (d) modified shape by Séquin.*

Conceptually, a freeform sweep can use any cross section. Figures 12a and 12c show two wood sculptures by Brent Collins [3], where the cross section consists of the union of two overlapping discs. Figures 12b and 12d show emulations and further developments of this theme, modeled in the Berkeley SLIDE environment and fabricated on a rapid prototyping machine. As shown, these models do not use any standard tubular stock that could be readily bent in the way depicted, and thus they do not offer a ready pathway for making large-scale sculptures. One might perhaps try to start with just one tube and bend it in the required way. To this primary tube one would then add a second tube from which about one sixth of its mantle has been cut away. However, it is not clear how well such a cut-open cylinder could be bent into the exact shape required to make a good fit to the primary tube. Such sweeps can, of course, readily form knotted structures as depicted by Figures 12c and d.

Figure 12: *Sweep surfaces: (a) Collins: "Muscularity;" (b) Séquin: "Galapagos-6." Knotted sweeps: (c) Collins: "Trefoil;" (d) Séquin: "Figure-8 Knot."*

8. Knotty Sculptures

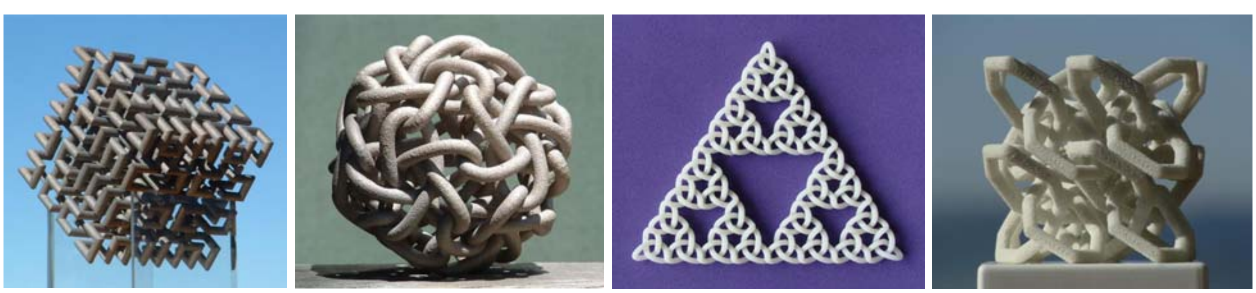

Figure 13: *Regular Knotty Sculptures: (a) "Hilbert Cube 512," (b) "DodecaPentafoil Tangle," (c) "Recursive Trefoil Knot 2D," (d) "Cubic Lattice of Figure-8 Knots."*

At other occasions I have talked extensively about *Knotty Sculptures* [16]. Conceptually most of my previously described knotty structures (Fig.13) belong to the family of tubular sculptures. But most of them were small-scale models that have been realized on different types of layered manufacturing machines, a process that is not amenable to fabricating large sculptures. However, these highly regular structures favor a modular approach. They each have just a few generic geometric elements, which could all be prefabricated individually and then assembled into the larger structure. In particular, "*Hilbert Cube 512*" (Fig.13a) only needs a single element: a 90° pipe elbow. The *Recursive Trefoil Knot 2D* also needs a small number of different elements (Fig.13c). The other two structures, however, may present some tough assembly problems, even though the number of individual types of elements is also rather low.

Figure 14: *Séquin: (a) "Chinese Button Knot" (copper tubing), (b) "Three Intertwined Loops," (c) "Chinese Button Knot" (drier duct), (d) "Glow-Worm."*

Another approach to make such knotty structures is to use some easily bendable material. Figure 14a shows a realization of the *Chinese Button Knot* (Knot 9_{40} in the standard knot tables) from thin copper tubing. For the three intertwined loops shown in Figure 14b, half-inch thick vinyl tubing has been used. The two structures shown in Figures 14c and 14d were realized with 6″-aluminum and 4″-vinyl drier duct; they depict again the *Chinese Button Knot* and a similar braided, but more complex 12-crossing knot (with internal lighting), respectively. A bigger challenge is posed by truly recursive knot structures such as the one presented by Robert Fathauer [6] where the pipe diameter is gradually reduced with subsequent recursion steps so as to allow arbitrarily many recursion steps without suffocating in the ensuing tangle.

9. Branching Tubular Assemblies

In the sculptures discussed so far, the assemblies of tubular elements followed individual poly-line paths or closed loops, and thus the mitering geometry at the "corners" was rather straightforward. But of course, tubular structures can have branches and may form much more complicated 3D graphs. If the tubular elements are skinny enough, then the details of the junctions escape the attention of the viewer; this is the case in Max Bill's *Construction with 30 Similar Elements* (Fig.15a) [12]. However, when tubes of larger diameter are joined, then careful attention has to be paid to the geometry of the mitered corners, as explained in Frank Smullin's keynote talk and demonstrated by the tutorial models shown in Figure 3b.

Figure 15: *(a) Bill: "Construction with 30 Similar Elements;" (b) Liberman: "Olympic Iliad;" (c) 'shade': "Steam pipe sculpture"; (d) Séquin: "Branching Drain Pipe."*

However, this kind of carefully calculated general mitering is rarely found in large-scale sculpture. Often large tubular elements are just left open-ended and connected to one another on their sides, as in Alexander Liberman's *Olympic Iliad* (Fig.15b). Another frequently found solution for creating tubular branches is to use the readily available "T"-junctions found in plumbing supply stores (Fig.15c) [18]. I have used more general and more complex junctions of three or more tubular elements in my *Branching Drain Pipe* (Fig.15d) and in my models of the projections of the 4-dimensional regular polytopes.

10. 3D Calligraphy

Figure 16: *(a) Serra: "Lead Piece;" (b) Hein: "Changing Neon Sculpture;" (c) RuBert: "Positronic Neural Net;" (d) Culbert: "SkyBlues."*

If the tubular material is thin and pliable enough, such as the lead pipe used by Richard Serra (Fig.16a), then it becomes a medium for "calligraphy" in 3-dimensional space. This becomes particularly apparent when neon tubes are used to form actual words as in large-scale advertisement and business logos. But neon tubes have frequently been used for purely artistic purposes. Since they can be turned on and off, they permit one to make dynamic time-varying sculptures, such as Jeppe Hein's *Changing Neon Sculpture* (Fig.16b), or Russ RuBert's *Positronic Neural Net* (Fig.16c) which changes in response to the viewers who walk around the sculptures. Neon tubes have also been used to make truly large-scale sculptures, such as Bill Culberts *SkyBlues* (Fig.16d), or Michael Hayden's installation at Chicago O'Hare airport [9].

11. Extensions to Organic Forms

Figure 17: *(a) 1994 Ad: "Absolut Scudera Vodka;" (b) Tunger: "Oh, Beautiful Life!" (c) Cragg: "Statue;" (d) Borofsky: "Dancers;" (e) Ochsner: "Libellotto".*

Most of the sculptures discussed so far have been completely abstract shapes. However, tubular elements can readily be used to approximate familiar natural shapes. Figure 17 shows a variety of sculptures that play with humanoid forms, ranging from advertisements of absolute vodka (Fig.17a) to John Tunger's celebration of dance (Fig.17b). Tony Cragg has taken the modular approach one step further with his *Statue* (Fig.17c) in which he uses a large number of aluminum manikins made from tubular elements to assemble a giant sculpture in the shape of two persons, thus introducing an element of recursion. As the tubular elements become more varied or are blended together more smoothly, the resulting humanoid forms can be made ever more life-like, as in Jonathan Borofsky's *Dancers* (Fig.17d). At this end of the spectrum, tubular sculptures seamlessly transition into purely free-form shapes, as exemplified by Claire Ochsner's *Libellotto* (Fig.17e).

12. Conclusions

Tubular elements are a surprisingly expressive and cost-effective medium. Assembling stock elements, such as segments of rods, bands, or tubes is a much less expensive way to make large durable metal or plastic sculptures, than casting individual free-form bronze pieces, welding them together, and then smoothing the surfaces across the various joints. CAD tools are a big help in the conception, detailed design, and aesthetic optimization of any sizeable sculpture, and they play a crucial role in creating the necessary shop drawings. Parameterized procedural descriptions of such geometries can turn this domain into a fertile playground for artistic experiments.

Acknowledgments

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References

[1] R. Berry, *Rising Wave*. (2009): http://baytrail.abag.ca.gov/vtour/map4/access/OystrBay/OystrBay.htm

[2] M. Bill, in: *aus der sammlung theo und elsa hotz*, Museum Jean Tinguely, Basel, (1998), pp 62-69.

[3] B. Collins, *Geometric Curvature and their Aesthetics*. Proc. Bridges, Winfield, Kansas, 2001, pp 81-88.

[4] J. de Rivera, *Construction # 35*. in: *Modern Sculpture*, by H. Read, Thames and Hudson, Inc., NYC, Fig. 274.

[5] Doggie Solutions, *Nobbly Wobblys*. (2009): http://www.doggiesolutions.co.uk/dog-throw-toy-4364-0.html

[6] R. Fathauer, *Fractale Knots Created by Iterative Substitution*, Proc. Bridges, Donostia, 2007, pp 335-342.

[7] *FUNtain Hydraulophone*. Ontario Science Centre webpage (2009): http://wearcam.org/oscfuntain/

[8] G. Hart, 72 Pencils, webpage (2009): http://www.georgehart.com/sculpture/pencils.html

[9] Michael Hayden: *Sky's the Limit*. (2009): flickr.com/photos/good_day/129662479/

[10] A. Holden, *Orderly Tangles*. Columbia University Press, New York, (1983).

[11] J. Huberman, Treklite Ink, webpage (2009): http://treklite.com/products/art/torus.htm

[12] E. Huettinger, *max bill*, abc edition, Zürich, (1978).

[13] M. Oldfield, *Tubular Bells*. (2009): http://tubular.net/

[14] C. O. Perry, *Ribbed Sculptures.* webpage (2009): http://www.charlesperry.com/Ribbed.html

[15] C. H. Séquin, *Patterns on the Genus-3 Klein Quartic*. Proc. Bridges, London, 2006, pp 245-254.

[16] C. H. Séquin, *Tangled Knots*. Proc. Art+Math=X, Intnl. Conf., Boulder CO, June 2-5, 2005, pp 161-165.

[17] C. H. Séquin and J. F. Hamlin, *The Regular 4-Dimensional 57-Cell*. SIGGRAPH'07, Sketches and Applications, San Diego, Aug. 4-9, 2007.

[18] 'shade', *Steam Pipe Sculpture.* (2009): http://www.steampunkmagazine.com/forum/viewtopic.php?t=404

[19] F. Smullin, *Analytic Constructivism: Computer-Aided Design and Construction of Tubular Sculptures*. Luncheon Presentation, 18th Design Automation Conf., Nashville, TN, June 30, 1981.

[20] J. Snoeyink, J. Stolfi, *Objects that cannot be taken apart with two hands*. Proc. of the 9th ACM Symp. on Computational Geometry (1993).