Simultaneous Coupling of Fluids and Deformable Bodies sketch_0566

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Introduction

We describe a method for simultaneous two-way coupling of fluid and deformable bodies. The interaction between a fluid and deformable body can create complex and interesting motion that would be difficult to convincingly animate by hand.

Previous approaches used a time splitting procedure that alternately fixes the fluid pressure while simulating the solid, then fixes the solids velocity while simulating the fluid. While this approach works reasonably well for physical systems with non-stiff coupling, it can lead to instability and visual artifacts for other systems. These problems occur because while solid velocities are fixed they will ignore arbitrarily large fluid pressures, and the converse when the fluid velocities are fixed. For a tightly coupled system like a piston, time splitting becomes untenable and difficulties can still arise even for less tightly coupled systems. Time splitting also requires non-physical fixes for closed systems. By enforcing simultaneous coupling our method avoids these artifacts and allows for substantially larger time steps.

Overview

The interaction between a fluid and a deformable solid occurs at the interface. The fluid applies pressure forces on solids' boundary and the solid imposes boundary fluxes on the fluid. This information exchange occurs simultaneously for real physical systems. Therefore, in order to simulate the complex interactions between a solid and a fluid we need to augment both simulations with extra degrees of freedom and create a combined fully coupled system. The key idea is that when performing pressure correction on the fluid and implicitly solving for the solid's node velocities, we need to simultaneously account for how fluid pressure changes will effect the deformable solid and how the solid's motion will effect the fluid.

Using a finite element or finite difference method and an implicit Newmark time integration scheme the dynamics of an elastic solid body deforming under pressure forces can be fully discretized in the following form:

$$\mathbf{A}u^{n+1} - \mathbf{J}p^{n+1} = b \tag{1}$$

where $\mathbf{A}{=}\frac{1}{h}\mathbf{M}{+}\mathbf{C}{+}\frac{h}{2}\mathbf{K}'$ and $b{=}\frac{1}{h}\mathbf{M}u^n{-}\frac{h}{2}\mathbf{K}'u^n{+}\mathbf{K}(d^n){+}f_e$. The solid's node displacement and velocity are denoted by d and u respectively. $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices. $\mathbf{K}$ is the non-linear stiffness matrix function and $\mathbf{K}'$ is it's tangent matrix evaluated at d^n. Forces due to the pressure of the fluid are computed and mapped by the $\mathbf{J}$ matrix. f_e are any additional external forces. Finally, h and n denote the timestep and time index respectively.

The fluid, on the other hand, needs to account for the boundary fluxes imposed by the solid while satisfying the incompressibility condition. The resulting Pressure Poisson equation can be formulated as:

$$-\mathbf{D_1}\mathbf{H}u^{n+1} + \frac{h}{q}\mathbf{D_2}\mathbf{G_2}p^{n+1} = \mathbf{D_2}v^* \tag{2}$$

where $\mathbf{D}_1$ and $\mathbf{D}_2$ are matrices for computing the divergence due to the boundary and internal fluid fluxes respectively.

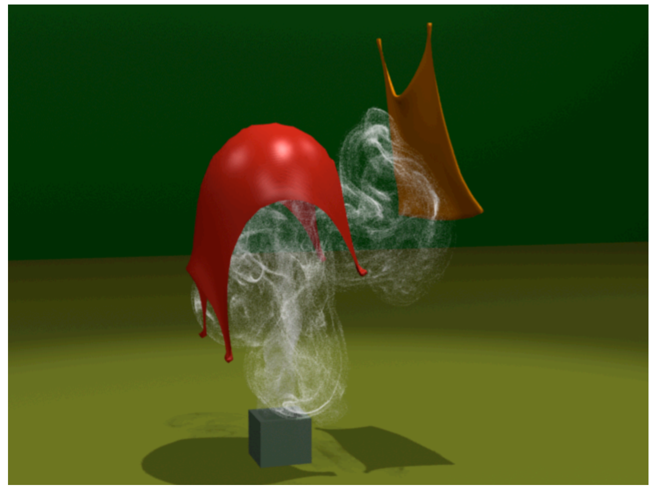

Figure 1: A frame from an animation of a jet interacting with two thin rubber sheets.

$\mathbf{G}_2$ is the gradient matrix for internal fluxes and $\mathbf{H}$ is a matrix that converts solid's boundary node velocities into fluxes on the fluids's boundary. Finally, v^* is the intermediate fluid velocity computed from the fluid simulation that needs to be projected onto it's divergence free component v^{n+1}.

In order to simulate full coupling between the fluid and the solid we combine Equation (1) and Equation (2) to solve for u^{n+1} and p^{n+1} simultaneously:

$$\begin{bmatrix} \mathbf{A} & -\mathbf{J} \\ -\mathbf{D}_1\mathbf{H} & \frac{h}{\rho}\mathbf{D}_2\mathbf{G}_2 \end{bmatrix} \begin{bmatrix} u^{n+1} \\ p^{n+1} \end{bmatrix} = \begin{bmatrix} b \\ \mathbf{D}_2 v^* \end{bmatrix} \tag{3}$$

Finally, we can compute the divergence free fluid velocities v^{n+1} as follows:

$$v_i^{n+1} = \begin{cases} (v^* - \frac{h}{\rho}(\mathbf{G}_2 p))_i & \text{if } i \text{ is not a boundary face} \\ (\mathbf{H}u^{n+1})_i & \text{if } i \text{ is a boundary face} \end{cases} \tag{4}$$

Results

Figure 1 shows a frame of animation demonstrating the interaction between a jet of smoke and two thin rubber sheets. The supplemental video includes this sheet example and also a comparison showing how simultaneous coupling can be stable while an otherwise identical time-spitting simulation will exhibit undesirable artifacts. With a time steps of $\frac{1}{30}$ sec the simultaneous coupling method presented in this paper remains stable while the time-splitting method goes unstable. The time-splitting method behaves stably only after reducing the time step to $\frac{1}{60}$ sec. Even though the resulting system in Equation (3) is larger than those used in time-splitting, it is still very sparse and by solving it our method does not incur an overhead large enough to offset the advantage of using large timesteps.